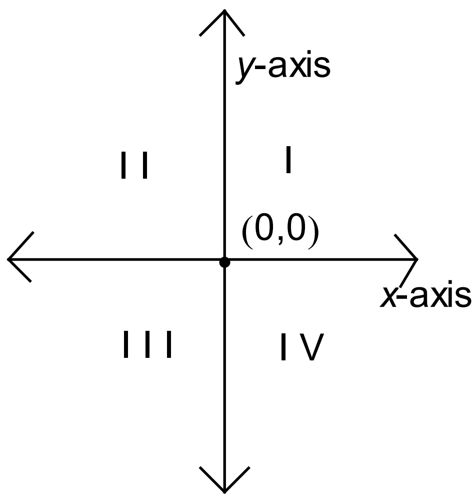
Section 4-1, Mathematics 104

Graphs

The Cartesian Plane

We draw a 2D graph by starting with a number line, and then adding a perpendicular number line.

We call the horizontal number line the ***x*-axis** and the vertical number line the ***y*-axis**.



These two lines divide the plane into 4 areas called **quadrants.**

The point where the two axes meet is called the **origin**.

Each point on the plane is described by a pair of numbers called an ordered pair. These numbers are called **coordinates.**

We always specify an x coordinate first (x,y).

The coordinates of the origin are (0,0).

These are called Cartesian coordinates.

They are named after Renee Descartes.

He's famous for saying, "I think therefore I am".

It used to be more common for mathematics and philosophy to mix.

Problem Solving Approaches

When given a problem there are often three ways to go about solving it.

1) **Numerical Approach**
There are a wide range of numerical methods, but the one we are talking about here is making a table of values. This is related to the **guess and check** method I previously discussed.

2) **Algebraic Approach**
This approach uses rules of algebra and its purpose is to find an exact answer in a methodical manner. We will learn a number of algebraic methods.

3) **Graphical Approach**
This is the "A picture is worth a 1000 words" method. In particular we are talking about drawing a Cartesian graph and leaning the solution from this picture. Like the numerical approach this is an inexact method. Unlike the numerical approach it's accuracy can depend on drawing skill.

Example:

$$6x-2y=4$$

We can put this in **functional form** by solving for y.

$$6x-2y=4$$
$$-2y=-6x+4$$
$$y=3x-2$$

We create a table of values.

x	y
-2	-8
-1	-5
0	-2
1	2
2	4

This gives us an idea given an x what to expect for a y.
Note that all the values ≥ 1 are positive and all the values ≤0 are negative

This can be a useful method to understand a problem when unsure of the nature of the solution, but in general this is not a very good way to a solution.

Verifying a solution

To verify whether a set of x,y values is a solution to an equation like $6x-2y=4$ you merely plug in the numbers and check whether the equality holds.

Example:

Which of these are solutions (7,20), (-6,-20)

$$6(7)-2(20)=42-40=2\neq 4$$
$$6(-6)-2(-20)=-36+40=4$$

This is always a good idea after you find a solution as a check.

Graphing a linear equation

The book is not wrong on this subject, but it is wrong.
You should remember from geometry that a line is determined by 2 points.
Therefore a linear equation only requires that you plot 2 points to know what the whole graph looks like.

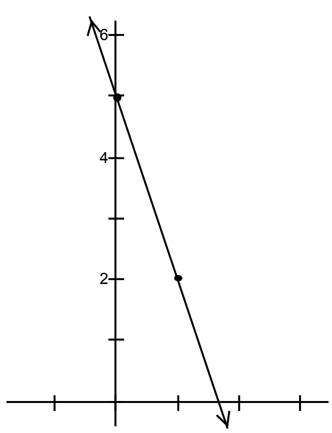
Example:

$$3x+y=5$$

Solving this we find

$$y=-3x+5$$

We can choose two easy points 0, and 1, giving solutions (0,5) and (1,2).
Plotting we see:



Note that the line passes through the y axis at the value 5.
This point (0,5) is known as the ***y* intercept.** We will using terminology for graphs other than straight lines.

An intercept is any point where a graph passes through an axis.
So we also have an x intercept.

Note that the x intercept is where y is zero and the y intercept is where x is zero.

Other types of graphs

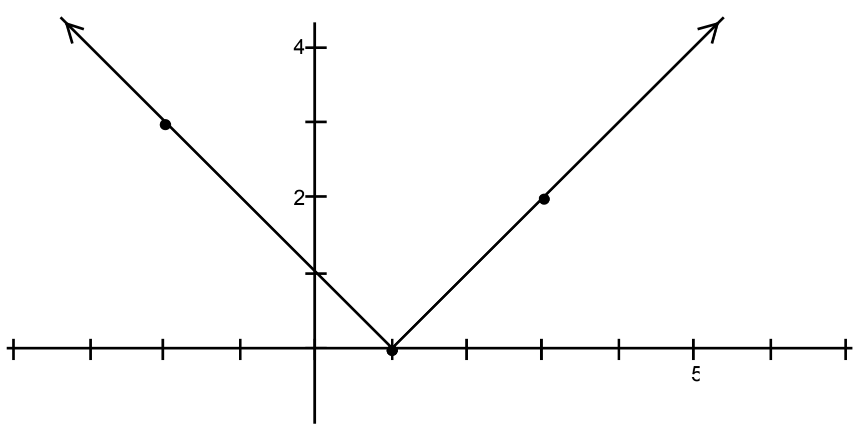
We may be called upon to graph an equation with an absolute value.

Example:

$$y = |x - 1|$$

We could create a table of values, but there is a more direct way.
Note that at x=1 something special happens. for $x \geq 1$ the value inside the absolute value brackets is always positive and for $x \leq 1$ the value inside the absolute value is always negative, and therefore is flipped. So something special will happen at x=1=> (1,0).

Let's pick two other points, on either side of 1, 2 and -2.



Note that we have two symmetric rays that begin at (1,0).
So to graph an equation with an absolute value, choose an x value that makes the absolute value zero, and then choose two points on either side.

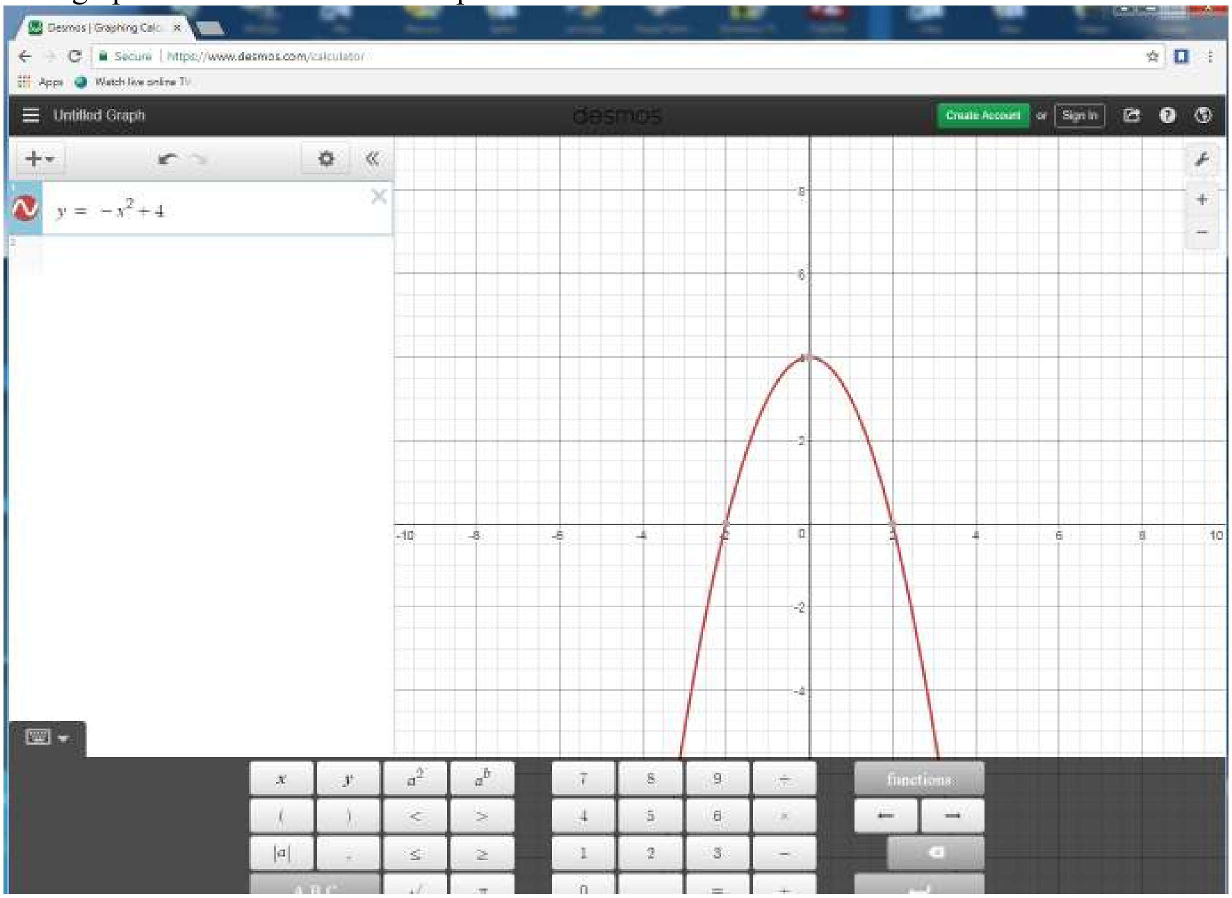
Non-Linear Graphs

Any graph where the variables have an exponent greater than 1 is a non-linear equation, which will have a graph that is not a line.

The book must have been written in the stone age because it ignores the technology that can assist.   Two examples are:

A graphing calculator such as the Ti 83.

Web graphic calculators such as https://www.desmos.com/calculator

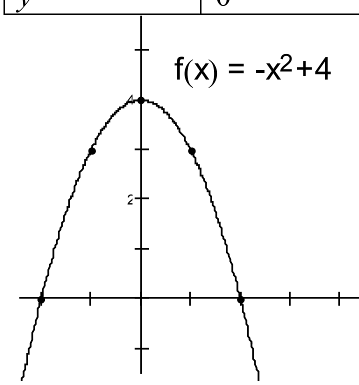


However if you are on a desert island with only the sand to make a graph with, it might help to make a table:

Example:

$y = -x^2 + 4$

x	-2	-1	0	1	2
y	0	3	4	3	0



This Shape is known as a Parabola.

This shape has many applications.

The reflector on most flashlights is a parabola, as are most telescopic mirrors.

It's also can be the trajectory of a comet that circles the sun once but never returns.

Finding Intercepts

A very practical skill when dealing with graphs is finding the intercepts.
This is really straight forward.

To find the y axis, plug zero into the equation for x and solve.
To find the x axis, plug zero into the equation for y and solve.

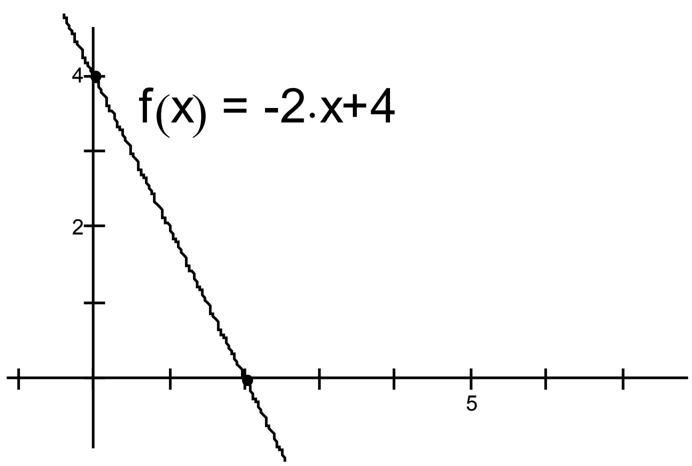
Example:

$$y = -2x + 4$$

To find the y intercept, plug in x=0 -> y=4

To find the x intercept, plug in y=0 -> 0 = -2x+4 so x=2.

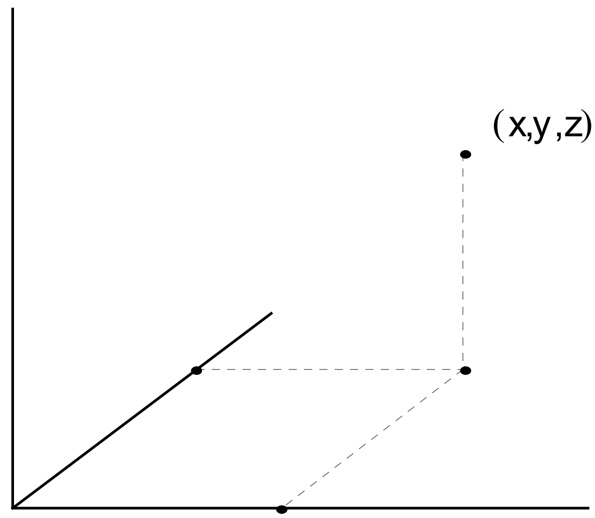
This is a good way to plot a linear equation since we now have two points (0,4) and (2,0)



Cool Stuff You don't need to know.

It seems coincidental that the number of coordinates is the same as the number of dimensions.

If we go to graphing in 3D this will seem to be the case.

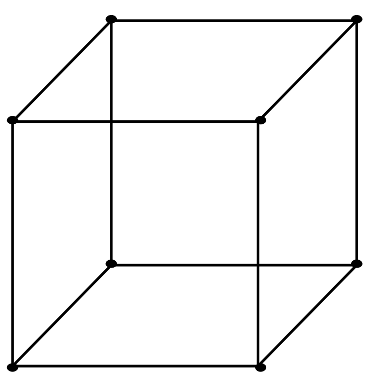


Note, now I am drawing a 3D representation on a 2D surface, so clearly you need your mind to see the depth involved.

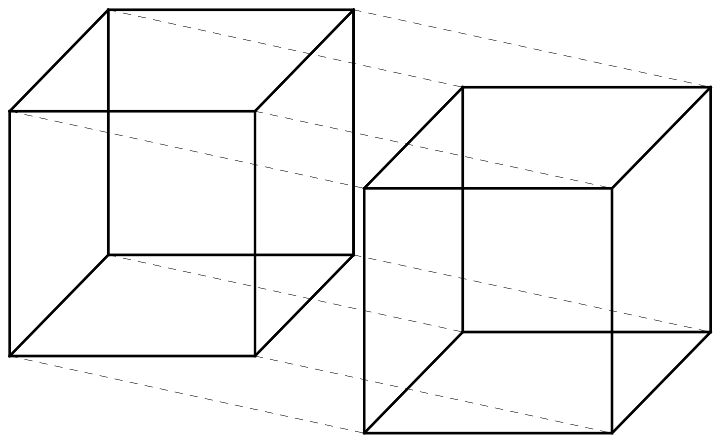
We can push this a little.

What would a 4D object look like?

We create a 3D cube by drawing two squares and connecting the vertices.



So if draw 2 cubes and connect the vertices we get something like this:



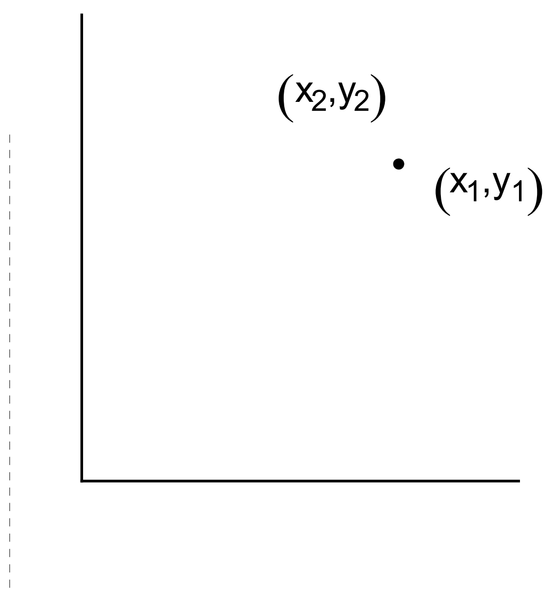
This 4D object has a name, it's called a tesseract.

This is not he only way to represent it.

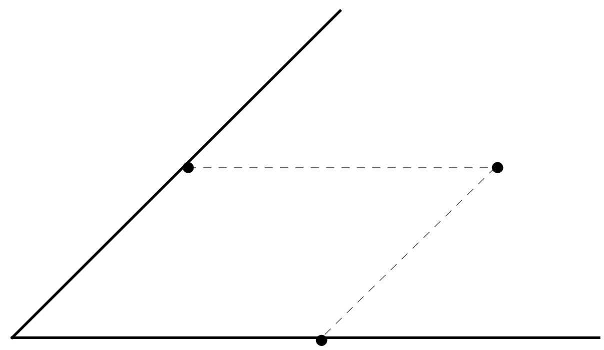
Back to 2D graphs for a second.

You should know that there is more than one way to use 2 numbers to represent a point on the graph.

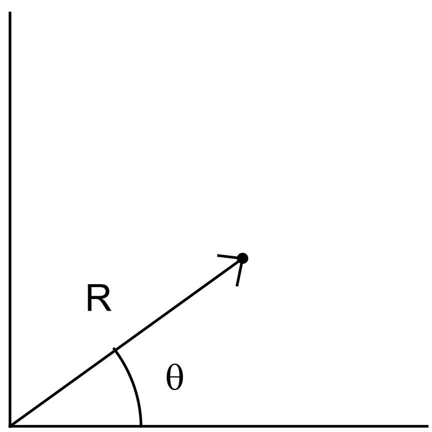
For one thing you can move the axis.



You can also use axes that are not perpendicular



And there are other possibilities:



This is what is known as a polar coordinates.

Going back to the question of whether we need 2 number to describe each point on the plane.

Actually we don't.

Let's say we have coordinates   (5,7)

We could just use the number 57

Then (123,456) would become 142536.

This works in 3D and onward.

This sounds very strange if you think about it.

It means that there is the same number of points on a plane or in all 3D space as there is on a line.

This is strange, but mathematically true.